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AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
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UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS 01730

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PREFACE

Atmospheric models are frequently generated for specific regions of the atmosphere in accordance with a particular set of observations. For example, for the region 20 to 90 km or for the region 120 to 100 km altitude. These models are not necessarily extended to sea level, neither are they designed to be continuous with some other existing model at either end of the included range. In some instances it becomes desirable to exactly connect two such models with a transition model atmosphere which for the examples cited would extend between 90 and 120 km. This report discusses a method suitable for generating such transition models.

ABSTRACT

Any set of temperature,pressure and density values which are realistic for one altitude can be exactly connected to another set of temperature, pressure and density values at a second altitude, with a model atmosphere defined by the appropriate linearly segmented two-layer temperature-altitude profile provided one of these layers is isothermal and the second is a nonzero constant gradient of temperature with respect to height.

The altitude comprising the intersection of the two segments of the required temperature-altitude profile is determined mathematically. The method for generating two-layer models is applied to the generation of three-layer and four-layer models.

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SECTION I

REVIEW AND EXTENSION OF THE METHOD FOR DEVELOPING TWO-LAYER TRANSITION MODELS

1. DEFINITION OF THE GENERAL TWO-LAYER TRANSITION MODELS AND THE SPECIFICATION OF THE PARTICULAR CLASS TO BE EXAMINED

Minzner (1966, pp. 5 and 6) has shown that, under a particular though not highly restrictive set of conditions, a two-layer transition model can be made to fit exactly between two sets of fixed boundary conditions. The first consists of a set of realistic values H_o, ρ_o and T_o, where H_o is the geopotential corresponding to one arbitrary geometric altitude, ρ_o is a corresponding realistic value of density, and T_o is a realistic value of the temperature parameter at H_o. The second consists of a set of realistic values H_n, ρ_n and T_n, where H_n is the geopotential corresponding to another arbitrary geometric altitude, ρ_n is a corresponding realistic value of density, and T_n is a corresponding realistic value of the temperature parameter. The subscript immediately to the right of the symbols H, ρ, or T is associated with the particular geopotential level to which these three quantities are related. In this discussion of transition models, the words <u>altitude</u> or <u>height</u> will be intended to imply the equivalent geopotential without such a statement necessarily being made.

The restrictive conditions defining the particular class of two-layer models to be considered are as follows:

(1) One of the two layers is characterized by a non-zero constant gradient of the temperature parameter relative to the geopotential such that this non-zero-gradient layer extends from H_o toward H_n for an unspecified altitude interval to the level H_{xo}.

(2) The second layer extending from H_{xo} to H_n, is characterized by a constant zero gradient of the temperature parameter relative to the geopotential. The symbol H_{xo} designates the altitude of the interface between the isothermal layer and the non-isothermal layer extending up to H_o.

The equations and computational procedures for generating the class of transition models as discussed in this report are developed on the basis of two further assumptions: (1) Geopotential H_n represents a lower altitude level than that represented by H_o such that the isothermal layer is the lower of the two layers of the transition model, and (2) the value of the temperature decreases with decreasing altitude from H_o to the isothermal layer. This class of model applies to situations similar to that portion of the U. S. Standard Atmosphere 1962 between altitudes of 117 to 79 kilometers.

The temperature parameter employed in the transition-model development is the molecular scale temperature which combines two variables, the kinetic

temperature and the mean molecular weight of the atmosphere, into a single variable in accordance with the following definition:

The molecular scale temperature, at any level H, is equal to the ratio of the kinetic temperature to the mean molecular weight, at that same level, times M_s the sea-level value of the molecular weight.

For simplicity, the notation commonly used for molecular scale temperature will be avoided in favor of T in this discussion of transition models, and the word temperature will imply molecular scale temperature.

2. DENSITY AND TEMPERATURE RELATIONSHIPS FOR THE SPECIFIED TYPE OF TWO-LAYER MODEL

In the type of model hypothesized, where an isothermal layer at temperature T_n extends upward from H_n to H_{xo} and a layer with a constant non-zero temperature-altitude gradient extends upward from H_{xo} to H_o, the density ρ_x at H_x may be expressed in terms of ρ_n and T_n by the following equations:

$$\rho_{xo} = \rho_n \exp \left\{ \frac{-(H_{xo} - H_n)\, Q}{T_n} \right\} \tag{1}$$

where

$$Q = \frac{G\, M_s}{R} = 0.0341631947 \,{}^{\circ}\mathrm{K/m'} \tag{2}$$

and

- G is the geopotential transformation coefficient equal to $9.80665\ \mathrm{m^2\ sec^{-2}\ (m')^{-1}}$
- M_s is the sea-level value of the molecular weight of air, 28.9644 kilograms per kilomole (kg/kmol)
- R is the universal gas constant 8.31432×10^3 joules $({}^{\circ}\mathrm{K})^{-1}$ $(\mathrm{kmol})^{-1}$

In such a model the density ρ_x at H_x may also be expressed relative to T_n and the upper-level boundary condition values, ρ_o, T_o, and H_o, which through H_x imply the value of the non-zero gradient of T between H_x and H_o. Thus,

$$\rho_{xo} = \rho_o \left[\frac{T_o}{T_n}\right]^{\left\{1 + \frac{(H_o - H_{xo})\, Q}{T_o - T_n}\right\}} \tag{3}$$

The value of H_{xo} is found by the simultaneous solution of Equations (1) and (3) as expressed by;

$$\rho_n \exp \frac{-(H_{xo} - H_n) Q}{T_n} = \rho_o \left[\frac{T_o}{T_n}\right]^{\left\{1 + \frac{(H_o - H_{xo}) Q}{T_o - T_n}\right\}} \tag{4}$$

The temperature-altitude gradient between H_{xo} and H_o specified by L_{xo} is given by

$$L_{xo} = \frac{T_o - T_n}{H_o - H_{xo}} \tag{5}$$

Here, the symbol for the gradient has two subscripts, the first designating the unknown lower end of the associated layer and the second designating the known upper end of the associated altitude interval.

3. ANALYTICAL EXPRESSION FOR THE GENERATION OF THE TWO-LAYER MODELS

Because it was not at first suspected that an analytical solution for H_{xo} in Equation (4) could be found, numerical methods involving digital computers were used to determine the value of H_{xo} for a number of transition atmospheres (Minzner, 1966, pp. 7 to 24). It may now be shown, however, that an analytical solution for H_{xo} in Equation (4) does exist.

Dividing both sides of Equation (4) by ρ_o and taking the natural logarithm of both sides of the resulting expression leads to

$$\left[1 + \frac{(H_o - H_{xo}) Q}{T_o - T_n}\right] \cdot \ell n \left[\frac{T_o}{T_n}\right] \Big\} - \frac{(H_n - H_{xo}) Q}{T_n} = \ell n \left[\frac{\rho_n}{\rho_o}\right] \tag{6}$$

The separation of the numerators of the two fractions containing H_x, so as to permit the extraction of H_{xo}, followed by a rearrangement of the terms leads to the expression

$$\frac{H_{xo} Q}{T_n} - \frac{H_{xo} Q}{T_o - T_n} \cdot \ell n \left[\frac{T_o}{T_n}\right] = \ell n \left[\frac{\rho_n}{\rho_o}\right] + \frac{H_n Q}{T_n} - \left[1 + \frac{H_o Q}{T_o - T_n}\right] \cdot \ell n \left[\frac{T_o}{T_n}\right] \tag{7}$$

Factoring H_{xo} out of the left-hand side of the equation and dividing both sides of that equation by the remainder of the left-hand side yields

$$H_{xo} = \frac{\ln\left[\dfrac{\rho_n}{\rho_o}\right] + \dfrac{H_n Q}{T_n} - \left[1 + \dfrac{H_o Q}{T_o - T_n}\right] \cdot \ln\left[\dfrac{T_o}{T_n}\right]}{\dfrac{Q}{T_n} - \dfrac{Q}{T_o - T_n} \cdot \ln\left[\dfrac{T_o}{T_n}\right]} \tag{8}$$

An evaluation of this expression using the boundary conditions of model Ar (Minzner 1966, Table 2, p. 13) yielded H_{xo} = 101.9646 km. This value agrees, to within one part in the sixth significant figure, with the value obtained by the numerical methods (Minzner, 1966, Table 3, p. 14), and thereby simultaneously substantiates the validity of the numerical method as well as of the analytical solution of H_{xo} given by Equation (8).

The subscript o to the right of x in H_{xo} is used to indicate that this particular value of H_x is calculated with the upper boundary conditions at level H_o. If, as will be seen in the generation of a three-layer model, H_x is calculated from some other upper-level boundary condition, as at H_m, the notation associated with x will be m as in H_{xm}.

The value H_{xo} from Equation (8), and the value L_{xo} from Equation (5), along with the upper and lower set of boundary conditions completely define the only two-layer transition model, of the type under consideration which satisfies the boundary conditions. There are multilayer models (three or more layers), however, which satisfy the same upper and lower set of boundary conditions and the unique two-layer model serves as a basis for the generation of these multilayer models. Computer Programs for the generation of two-layer models are given in Appendix A.

SECTION II

GENERATION OF THREE-LAYER MODELS

The method for the generation of a two-layer model between upper and lower boundary conditions can be extended to the generation of an infinite number of three-layer models. The method consists essentially of arbitrarily selecting an altitude interval for the uppermost layer, computing the conditions at the base of that layer from the initial upper-boundary conditions, and using these new values as the basis for the computation of the related two-layer model. The detailed steps to be used are as follows:

(1) Compute the value of H_{xo} and L_{xo} for that two-layer model which fits the initial boundary conditions.

(2) Select a suitable value H_1 as the base of the uppermost or first layer of the desired three-layer model. A suitable value meets the condition $H_{xo} < H_1 < H_o$. An infinite number of possible values of H_1 exist between H_{xo} and H_o.

(3) Select a suitable value of L_1 for the layer H_o to H_1. It may be demonstrated that if the gradients of the several layers are to decrease monitonically from H_o to H_n, the value of L_1 must meet the condition $L_{xo} < L_1$.

Furthermore, it will be seen that L_1 must meet the condition $L_1 < L_{max}$ where L_{max} is an upper limit associated with T_{1min} at H_1 as in Figure 1. The determination as to whether $L_1 < L_{max}$ is based upon the result of a test following a later step in the proceedure. At this point, however, a trial value of L_1 is chosen. An infinite number of possible values of L_1 meet the necessary condition $L_{xo} < L_1 < L_{max}$.

(4) Compute T_1 at the selected value H_1 from the selected value L_1 using the relationship

$$T_1 = T_o - L_1 (H_o - H_1) \tag{9}$$

(5) Compute the value of ρ_1, the density at H_1, using the relationship

$$\rho_1 = \rho_o \left[\frac{T_o}{T_1}\right]^{\left\{1 + \frac{(H_o - H_1)\, Q}{T_o - T_1}\right\}} \tag{10}$$

The set of values H_1, T_1, and P_1 serve as a new upper-boundary condition which with H_n, T_n, and P_n provide the means for generating a two-layer model between H_1 and H_n, where these two layers are designated layers 2 and 3 of the three-layer model. The interface between layers 2 and 3 is designated H_{x1}.

(6) Determine H_{x1} using Equation (8) with suitable subscript modifications, i.e., subscript zero is changed to one. If L_1 has been properly selected in step 3, H_{x1} will be found to have a value such that $H_n < H_{x1} < H_{xo}$, and this condition serves as the test for suitability of L_1.

(7) Compute the gradient for the layer between H_{x1} and H_1 by using Equation (5) with the subscript zero increased to subscript 1.

The conditions for the test of suitability of L_1 based on the value of H_{x1} have been developed through the following argument:

It is apparent that if L_1 is selected to be equal to L_{xo}, H_1 is not really the base of a new layer, but is a level within the layer H_{xo} to H_o. Computations based on H_1, T_1, and P_1 would then yield $H_{x1} = H_{xo}$, and one would have determined only a redundant case of the original two-layer model. As the value of L_1 is allowed to increase from L_{xo}, for a fixed value of H_1, the value of H_{x1} progressively decreases from H_{xo} until, in the limit for $L_1 = L_{max}$, the value of H_{x1} becomes identically H_n in which case the isothermal layer has vanished as indicated in Figure 1. For values of $L_1 > L_{max}$, the computed values of H_{x1} become less than H_n, indicating an impossible situation.

It is also possible to select values of L_1 such that $L_{min} < L_1 < L_{xo}$. As L_1 is decreased from L_{xo}, the value of H_{x1} increases from H_{xo} until for L_{min}, $H_{x1} = H_1$. For this case, layer 2 has the impossible conditions of zero thickness and an infinite temperature gradient. All values of $L_1 < L_{xo}$ are impossible however, if the model is constrained to one having a monitonically decreasing set of values of L for successive layers. It follows therefore, that the only suitable values of L_1 are those which lead to values of H_{x1} such that $H_n < H_{x1} < H_{xo}$. An infinite number of such values exist.

If values of L_1 are constrained to be integral multiples of some fixed values ΔL_1, and if they are constrained to be within the range L_{xo} to L_{max}, the smallest suitable values of L_1 may be expressed as

$$L_{1a} = j_1 \Delta L_1 \tag{11}$$

where j_1 satisfies the inequality

$$\frac{L_{xo}}{\Delta L_1} < j_1 \leq 1 + \frac{L_{xo}}{\Delta L_1} \tag{12}$$

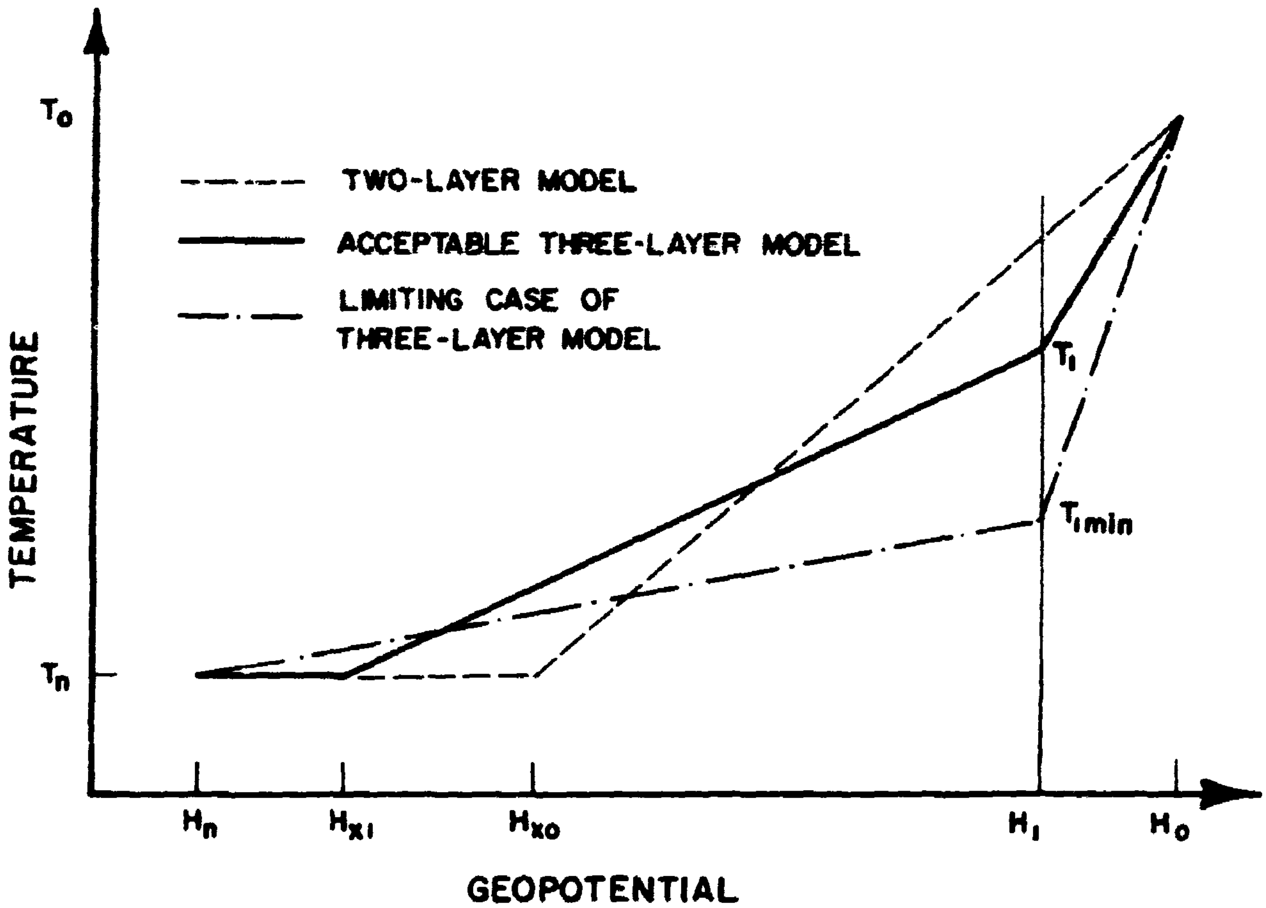

Figure 1. Schematic temperature-altitude diagram for a two-layer model atmosphere, for an acceptable related three-layer model, and for a limiting case of the three-layer models.

The series of acceptable values of L_1 and the related series of values of H_{x1} are then

$$L_{1a} = (j_1 + 1 - 1)\,\Delta L_1\,, \qquad H_{x1a} \tag{13a}$$

$$L_{1b} = (j_1 + 2 - 1)\,\Delta L_1\,, \qquad H_{x1b} \tag{13b}$$

$$L_{1c} = (j_1 + 3 - 1)\,\Delta L_1\,, \qquad H_{x1c} \tag{13c}$$

etc., until a value $L_{1\delta}$ is reached for which $H_{x1\delta}$ is less than H_n. This situation is shown schematically in Figure 2.

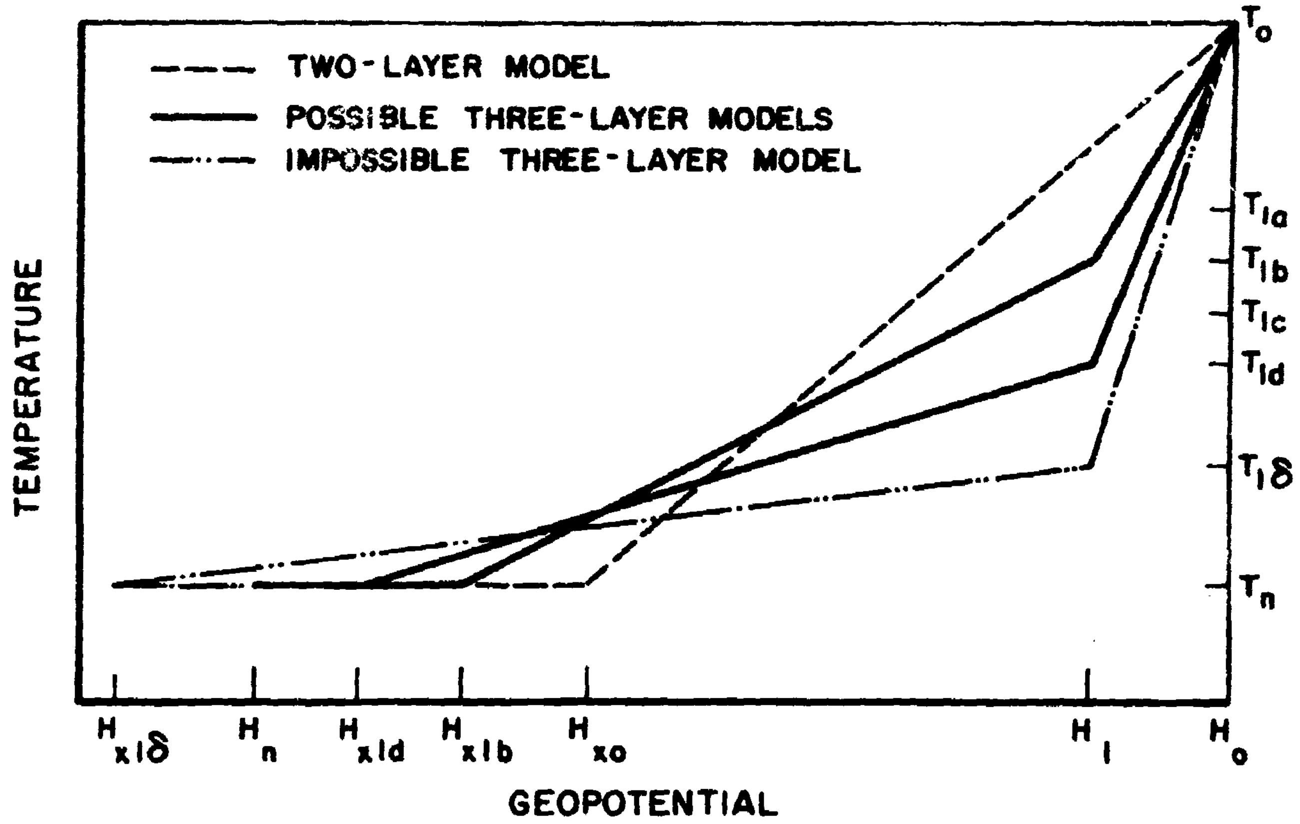

Figure 2. A schematic temperature-altitude diagram showing four possible three-layer models and one impossible three-layer model.

SECTION III

GENERATION OF MULTILAYER MODELS

The above procedure may be extended to the generation of four-layer models and on to the generation of n-layer models. The procedure consists of generating a three-layer model as indicated above, and then taking the lower two layers of this model, follow the procedure of steps 2 through 7 of Section II, but with subscript 1 replaced by subscript 2, and subscript o replaced by subscript 1. In a similar manner, the procedure may be extended to a model of any number of layers.

The altitudes specifying the boundaries of the four layers of each of a set of several possible four-layer models for a specified set of boundary conditions are given in Table 1.

TABLE 1

DEFINITIONS OF POSSIBLE FOUR-LAYER MODELS
CONSISTENT WITH THE BOUNDARY CONDITIONS
RHOO= .246100E-07 KG/M3, TO= 382.244 DEG K AT HO= 117776.0 M
RHON= .198200E-04 KG/M3, TN= 190.650 DEG K AT HN= 79000.0 M
AND WITH THE ASSIGNED VALUES OF H AND DL,
H1 = 110000.0 DL = .100000E-02
H2 = 105000.0 DL = .100000E-02

MINIMUM HEIGHT FOR BASE OF LAYER 1 IS HXO= 100503.0 M.
MINIMUM GRADIENT FOR LAYER 1 IS GLXO .110921E-01 DEG/M.

SET-01 MODELS DEFINED BY LAYER-1 GRADIENT OF 12 DEG/KM.

MINIMUM HEIGHT FOR BASE OF LAYER 2 IS HX1= 99731.2 M.
MINIMUM GRADIENT FOR LAYER 2 IS GLX1= .957098E-02 DEG/M.

MODEL 1 OF SET 01 DEFINED BY LAYER-2 AND -1 GRADIENTS, 10 AND 12 DEG/KM.

LEVEL	H METERS	RHO KG/M3	T DEG K	L DEG/M
0	117776.0	.246100E-07	382.244	
1	110000.0	.722249E-07	288.932	.120000E-01
2	105000.0	.167158E-06	238.932	.100000E-01
3	99387.0	.513486E-06	190.650	.860190E-02
4	79000.0	.198200E-04	190.650	.000000E+00

MODEL 2 OF SET 01 DEFINED BY LAYER-2 AND -1 GRADIENTS, 11 AND 12 DEG/KM.

LEVEL	H METERS	RHO KG/M3	T DEG K	L DEG/M
0	117776.0	.246100E-07	382.244	
1	110000.0	.722249E-07	288.932	.120000E-01
2	105000.0	.171873E-06	233.932	.110000E-01
3	98452.7	.607072E-06	190.650	.661069E-02
4	79000.0	.198200E-04	190.650	.000000E+00

MODEL 3 OF SET 01 DEFINED BY LAYER-2 AND -1 GRADIENTS, 12 AND 12 DEG/KM.

LEVEL	H METERS	RHO KG/M3	T DEG K	L DEG/M
0	117776.0	.246100E-07	382.244	
1	110000.0	.722249E-07	288.932	.120000E-01
2	105000.0	.176835E-06	228.932	.120000E-01
3	97275.3	.749666E-06	190.650	.495581E-02
4	79000.0	.198200E-04	190.650	.000000E+00

REFERENCES

Minzner, R. A., 1966: "Studies of Atmospheric Structure and Variability of the Earth's Atmosphere," GCA Technical Report No. 66-14-N, Final Report, Contract No. NASW-1225.

APPENDIX A

Program 1

The program for defining a two-layer transition model atmosphere by the method of successive approximation and from the definition computing the various properties as indicated in Tables 3 through 10 of the final report on Contract No. NASW-1225 (Minzner, 1966).

PROGRAM 1

```
C     SOURCE DECK FOR THE COMPUTATION OF TWO-LAYER TRANSITION MODEL
C     ATMOSPHERES BETWEEN UPPER LAYER AND LOWER LAYER BOUNDARY CONDITIONS
C     WHEN THE FOLLOWING CONDITIONS REGARDING TEMPERATURE-ALTITUDE
C     GRADIENT PREVAILS
C     1. IT HAS A DIFFERENT CONSTANT VALUE FOR EACH LAYER.
C     2. IT HAS A POSITIVE VALUE FOR THE UPPERMOST LAYER.
C     3. IT HAS A ZERO VALUE FOR THE LOWEST LAYER.
C     THE METHOD INVOLVES THE SIMULTANIOUS NUMERICAL SOLUTION OF
C     TWO EQUATIONS USING THE METHOD OF SUCCESSIVE APPROXIMATIONS
C     AS DISCUSSED BY MINZNER IN GCA TECH RPT. NO.66-14-N
C     JUNE 1965. REVISED MARCH 1966
   13 READ 101,DEG,HB,HPB,PHB,HA,HPA,PHA,FMON,T1,T2
C                                 READ IN INITIAL CONDITIONS FOR MODEL
C                                 T1 AND T2 ARE THE TITLE FOR THE MODEL
  101 FORMAT(A3,F4.0,F10.7,F11.4,F9.4,F9.5,F10.3,1XA4,2A5)
      READ 115,FL,HBB
  115 FORMAT(F4.1,1XF5.0)
      E=2.71828183
      EPSI=.1E-08
      X=HB+.1
      PUNCH 112
  112 FORMAT(35HPROPERTIES OF TRANSITION ATMOSPHERE)
      PUNCH 106,T1,T2,DEG,FMON
  106 FORMAT(2A5,2X1H(,A3,1H),2XA4,4X17HTO JACCHIA MODELS/)
C                                 SOLVE THE TWO EQUATIONS SIMULTANEOUSLY USING
C                                 THE METHOD OF SUCCESSIVE APPROXIMATIONS
    2 BETA=(HA-X)/(HPA-HPB)
      Y=(1.+BETA)*LOG(HPA/HPB)+LOG(PHA)
      XOLD=X
      X=-(Y-LOG(PHB))*HPB/LOG(E)+HB
      IF(SENSE SWITCH 1) 4,3
C                                 SENSE SWITCH 1 ON WILL LET THE OPERATOR
C                                 WATCH THE DIRECTON THE X VALUE IS
C                                 GOING IN.
    3 PRINT 103,X
  103 FORMAT(F14.8)
    4 IF(ABS(XOLD-X)-EPSI)1,1,2
    1 CONTINUE
      Y=EXP(Y)
      BETA=1./BETA
      XPT=X+0.
      PUNCH 102,DEG,FMON,X,Y
  102 FORMAT(A3,1XA4,2X3HX =F10.5,2X2HKM4X7HDEN X =E14.8,2X4HG/M3)
      PUNCH 113,HA,HPA,HPB
  113 FORMAT(19HGEOPOT SCALE HT AT F9.4,1X4HKM =F9.5,11H, AT X KM =,
     1F10.7)
      PUNCH 109,BETA
  109 FORMAT(35HGRADIENT OF GEOP SCALE HT ABOVE X =E14.8)
      PUNCH 114,FL,HBB,HB
  114 FORMAT(23HGRADIENT OF MOL. TEMP. ,F4.1,10H FOR BTWN ,F5.0,4H TO ,F4.0)
      PUNCH 110,HB
  110 FORMAT(38HGRADIENT OF MOL. TEMP. ZERO FOR H BTWN,F4.0,6H AND X)
      PUNCH 107
  107 FORMAT(3X11HALTITUDE KM10X15HTEMP DEG KELVIN4X6HMOL WT2X,
```

```
     112HDENSITY G/M34X11HSCALE HT KM)
      PUNCH 108
  108 FORMAT(1X6HGEOMET4X6HGEOPOT7X4HMOL.3X7HKINETIC28X6HGEOMET1X,
      RHOX=Y
      READ 104,DEG,R,GPHI
  104 FORMAT(A3,F9.0,F8.5)
      H=XPT
      Z=R*H/(GPHI*R/9.80665-H)
C                                Z IS THE GEOMETRIC ALTITUDE
      RHOH=PHA*(HPA/(HPA+BETA*(H-HA)))**((1.+BETA)/BETA)
C                                RHOH IS THE DENSITY
      HH=(HPB+BETA*(H-X))*9.80665/GPHI*((R+Z)/R)**2
C                                HH IS THE SCALE HEIGHT
      TM=34.1632*(HPB+BETA*(H-X))
      T=TM*(1.-2.375811E-03*(Z-90.))
C                                T IS THE TEMPERATURE
      HPH=HPB+BETA*(H-X)
      SM=28.9644-.0688133*(Z-90.)
C                                SM IS THE MOLECULAR WEIGHT
      TM=TM+.005
      T=T+.005
      SM=SM+.005
      HH=HH+.0005
      HPH=HPH+.0005
C                                ROUND THE VALUES
      PUNCH 105,Z,H,TM,T,SM,RHOH,HH,HPH
C                                PUNCH DATA CARD
  105 FORMAT(F8.3,2XF8.3,3XF7.2,3XF7.2,3XF6.2,2XF12.5,2XF7.3,2XF7.3)
     16HGEOPOT/)
C                                METHOD FOR FINDING NEXT EVEN KM.
      XNEW=DRH(X)
      XNEW=XNEW/2.
      XNEW=DRH(XNEW)
      XNEW=(XNEW*2.)+2.
      XP=X*10.
      XP=DRH(XP)/10.
      IF(XNEW-XP-2.) 8,9,8
    9 H=XP
      GO TO 7
    8 H=XNEW
    7 IF(H-XPT)5,5,23
   23 Z=R*H/(GPHI*R/9.80665-H)
      RHOH=PHA*(HPA/(HPA+BETA*(H-HA)))**((1.+BETA)/BETA)
      HH=(HPB+BETA*(H-X))*9.80665/GPHI*((R+Z)/R)**2
      TM=34.1632*(HPB+BETA*(H-X))
      T=TM*(1.-2.375811E-03*(Z-90.))
      HPH=HPB+BETA*(H-X)
      SM=28.9644-.0688133*(Z-90.)
      FLM=34.1632*BETA
C                                FLM IS THE GRADIENT
      TM=TM+.005
      T=T+.005
      SM=SM+.005
      HH=HH+.0005
```

```
      HPH=HPH+.0005
      IF(Z-120.)40,6,6
   40 PUNCH 105,Z,H,TM,T,SM,RHOH,HH,HPH
    5 H=H+2.
      GO TO 7
    6 Z=R*X/(GPHI*R/9.80665-X)
      Z=DRH(Z)/2.
      Z=DRH(Z)*2.+2.
   12 H=R*Z/(R+Z)*GPHI/9.80665
      RHOH=PHA*(HPA/(HPA+BETA*(H-HA)))**((1.+BETA)/BETA)
      SM=28.9644-.0688133*(Z-90.)
      HH=(HPB+BETA*(H-X))*9.80665/GPHI*((R+Z)/R)**2
      TM=34.1632*(HPB+BETA*(H-X))
      T=TM*(1.-2.375811E-03*(Z-90.))
      FLM=34.1632*BETA
      HPH=HPB+BETA*(H-X)
      TM=TM+.005
      T=T+.005
      SM=SM+.005
      HH=HH+.0005
      HPH=HPH+.0005
      PUNCH 105,Z,H,TM,T,SM,RHOH,HH,HPH
      IF(Z-120.)10,11,11
   10 Z=Z+2.
      GO TO 12
   11 H=HB
      X=HB
      BETA=0.
   18 Z=R*H/(GPHI*R/9.80665-H)
      HH=(HPB+BETA*(H-X))*9.80665/GPHI*((R+Z)/R)**2
      TM=34.1632*(HPB+BETA*(H-X))
      IF(Z-90.)26,26,27
   26 T=TM
      SM=28.96
      GO TO 28
   27 T=TM*(1.-2.375811E-03*(Z-90.))
      SM=28.9644-.0688133*(Z-90.)
   28 RHOH=PHB*EXP(-(H-HB)/HPB)
      HPH=HPB+BETA*(H-X)
      XPRIM=DRH(XPT)
      TM=TM+.005
      T=T+.005
      SM=SM+.005
      HH=HH+.0005
      HPH=HPH+.0005
      IF(H-XPRIM)14,14,15
   14 PUNCH 105,Z,H,TM,T,SM,RHOH,HH,HPH
      IF(H-HB)16,41,16
   41 IF(SENSE SWITCH 2)16,17
C                                SENSE SWITCH 2 ON. INCREASE THE HEIGHT BY 2.
C                                SENSE SWITCH 2 OFF, INCREASE THE HEIGHT BY 1.
   17 H=H+1.
      GO TO 18
   16 H=H+2.
```

```
      GO TO 18
   15 Z=R*X/(GPHI*R/9.80665-X)
      Z=DRH(Z)/2.
      Z=DRH(Z)*2.+2.
   21 H=R*Z/(R+Z)*GPHI/9.80665
      IF(H-HB)25,25,22
   22 RHOH=PHB*EXP(-(H-HB)/HPB)
      HH=(HPB+BETA*(H-X))*9.80665/GPHI*((R+Z)/R)**2
      TM=34.1632*(HPB+BETA*(H-X))
      IF(Z-90.)29,29,30
   29 T=TM
      SM=28.96
      GO TO 31
   30 T=TM*(1.-2.375811E-03*(Z-90.))
      SM=28.9644-.0688133*(Z-90.)
   31 HPH=HPB+BETA*(H-X)
      TM=TM+.005
      T=T+.005
      SM=SM+.005
      HH=HH+.0005
      HPH=HPH+.0005
      IF(H-XPT)19,20,20
   19 PUNCH 105,Z,H,TM,T,SM,RHOH,HH,HPH
   25 Z=Z+2.
      GO TO 21
   20 PUNCH 111,FLM,HA
  111 FORMAT(23HGRADIENT OF MOL. TEMP. F10.5,2X19HDEG/KM FOR H BTWN X,
     14H AND,F9.4//)
      GO TO 13
      END
```

PROGRAM 2

The program for defining a two-layer model by the evaluation of an algebraic function, which replaces the successive approximation method.

PROGRAM 2

```
C     SOURCE DECK FOR TWO-LAYER TRANSITION MODEL ATMOSPHERE
C     BETWEEN UPPER-LEVEL AND LOWER-LEVEL BOUNDARY CONDITIONS
C     WHEN THE FOLLOWING CONDITIONS REGARDING TEMPERATURE-ALTITUDE
C     GRADIENT PREVAILS
C     1. IT HAS A DIFFERENT CONSTANT VALUE FOR EACH LAYER.
C     2. IT HAS A POSITIVE VALUE FOR THE UPPERMOST LAYER.
C     3. IT HAS A ZERO VALUE FOR THE LOWER LAYER.
  100 FORMAT(F10.1,2XE12.6,1XF10.3,2XF10.1,2XE12.6,1XF10.3)
   51 READ 100,HN,RHON,TN,HO,RHOO,TO
      BEGIN TRACE
      Q=3.4163195E-02
      HXO=(LOGF(RHON/RHOO)+HN*Q/TN-(1.+HO*Q/(TO-TN))*LOGF(TO/TN))
      HXO=HXO/(Q/TN-(Q/(TO-TN))*LOGF(TO/TN))
      GLXO=(TO-TN)/(HO-HXO)
      RHOXO=RHON*EXPF(-Q*(HXO-HN)/TN)
  120 FORMAT(21X33HDEFINITION OF THE TWO-LAYER MODEL)
  111 FORMAT(1H )
  112 FORMAT(10X5HLEVEL,5X1HH,11X3HRHO,11X1HT,11X1HL)
  113 FORMAT(18X43HMETERS        KG/M3         DEG K        DEG/M)
  114 FORMAT(10XI3,2XF10.1,2XE12.6,1XF10.3,2XE12.6)
  115 FORMAT(10XI3,2XF10.1,2XE12.6,1XF10.3,3X11H.000000E+00)
      PUNCH 111
      PUNCH 111
      PUNCH 120
      PUNCH 111
      PUNCH 112
      PUNCH 113
      PUNCH 111
      LEVEL=0
      PUNCH 114,LEVEL,HO,RHOO,TO
      LEVEL=LEVEL+1
      PUNCH 114,LEVEL,HXO,RHOXO,TN,GLXO
      LEVEL=LEVEL+1
      PUNCH 115,LEVEL,HN,RHON,TN
      END TRACE
      END
```